THE SOLUTION OF ALL Sphærical Triangles BOTH RIGHT AND OBLIQUE *By the Planisphare:*

Whereby two of the Sphærical partes sought, are at one position most easily found out.

Published with consent of the Author,

BY

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TO

My Worthy good Friend

Mr ROGER WILLIAMS.

Honoured Sir,

I *Have heard that the no leſſe Noble then Learned Maſter in Aſtronomy* Tycho Brahe *was wont to worke his computation of* ſphærical *triangles upon a globe of braſſe of three foot diameter, by meaſuring and deſcribing them thereon. And with the like facility and artifice I have oftentimes ſeen my Reverend friend* Mr W. O. *in his reſolution of all* ſphæricall *triangles both right and oblique, to uſe a planiſphære, without the tedious labour of Trigonometry by the ordinary Canons: which planiſphære he had delineated with his own hands, and uſed in his calculations more then Forty years before. I therefore ſeeing the expeditneſſe of that his operation, and relying on his accuſtomed readineſſe in doing good, made bold to intreat of him a copy of his*

Rules and way that he observed therein: which my request he courteously yielding to, added this also, that, if I should see good I might make them publique by setting out both the Rules and Instrument it self, for the use and benefit of such as are Students and Practicers in Astronomine. *Now knowing well the loving affection and worthy respect the Author beareth unto your self, I thought I could not doe a thing more acceptable to him, then to publish the same under your name, and thereby make known to the world, both the kind love and friendship between you and the Author, and also the honour which I worthily bear to you, and the desire I have to be accounted*

your obliged Servant

CHRISTOPHER BROOKES.

Wadham Coll.
Oxford May 25th
1651.

The ſolution of all Sphærical Triangles both right and oblique by the Planiſphære : whereby two of the Sphærical parts ſought, are at one poſition moſt eaſily found out.

THE FIRST PART, Of Right-angled ſpherical triangles.

1. THE right-angled ſphæricall triangle to be reſolved I ſignify ſtill by the letters A B C: ſetting A at the right angle, and B & C at the other two acute angles. So ſhall the baſe be B A, the cathetus C A, the hypotenuſe B C, the angle

at the base B, and the angle at the Cathetus C.

2. There are therefore besides the right angle (which is evermore the same) five circular parts, namely three sides, and two angles: of which, fower come into the account at once: two of them are given, and the other two found out.

3. *If the fower parts which at once come into the account be the three sides, and one angle*; let that acute angle be noted evermore with B: and the triangle may be resolved thus: Set the angle B at the center, reconing it upon the limb from the diameter; and the Base B A upon the diameter from the center; and the Cathetus C A upon the great circles from the diameter, by help of the parallels; and the Hypotenuse B C upon the Index from the

center. Note that if any of those accounts fall not just upon some line in the Instrument, either of the great circles or parallels, the excesse is to be æstimated in minutes or parts of a degree.

For *example* in a right-angled triangle sphærical, if the angle at the base be 54° 25′; and the Cathetus 38° 30′: Recon on the limb from the diameter 54° 25′, for the angle B, and to the end thereof set either of the legs of the Index: againe upon the limb from the diameter recon 38° 30′ for the Cathetus C A; and from the end of that arch æstimate reasonably a parallel circle till it meet with the Index: for that poynt of interfection shall shew both the Hypotenuse B C, 49° 57′ upon the Index: and the Base B A 34° 41′; upon the great circle meeting also in that poynt.

Lastly to find the angle C, take C A for the Base, and B A for the Cathetus; and to the end of that Cathetus apply the Index: so shall you have the angle sought for C upon the limb, 48° 01′.

4. Otherwise if you please, you may set the angle B at the pole, reconed upon the diameter from the limb: and B A upon the limb from the pole: and C A upon the Index from the limb: and B C upon a great circle from the pole.

5. But *if the fower parts which at once come into the account, be the two oblique angles, and the two right sides*: the triangle may be resolved thus. Recon the lesser angle on the Index from the center: and the greater angle on a great circle from the pole: so both of these with the axis, shall include a quadrantal triangle: and the greater right side shall be on the limb from the pole: and the lesser right side shall be upon the diameter from the center.

Lastly two of the fower parts being had, there will be no difficulty in finding out the Hypotenuse.

6. Now therefore having shewed, in the Rule and Example of sect: 3, how every one of those five circular parts are placed on the Instrument; I shall not need hereafter in solution or working of any right-angled or quadrantal triangle proposed to expresse those parts in words at length, but only to name them by their proper letters, B, BC, BA, CA, & C.

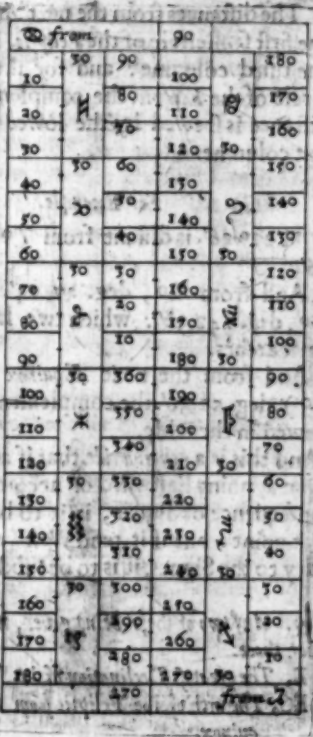
7. *To convert Howres with decimal parts, into degrees of the Æquinoctiel with decimal parts*; Multiply the Howers by 5, and the product thereof by 3.

Contrariwise *to convert degrees of the Æquinoctiel into Howers*; divide the degrees by 3, and the quotient thereof by 5. Thus Hor 13,472 = Deg 202,08.

8. A table for the distances of ever degree of the *Ecliptic* from the fower cardinall points.

This table consists of fix columnes: the fower middle columnes shew the Longitude of every fift degree of the *Ecliptic* from ♈o. whereby also the rest of the degrees may be æstimated.

The two outmost columnes shew the distance from the two Solsti-



cies, namely the first columne from ♋ 0, and the sixt from ♑ 0.

The distances from the next *Æquinox*, for the first semicircle of the *Ecliptic*, is shewed by the third columne: and for the later semicircle of the *Ecliptic*, the complement of that distance is shewed by the lower halfe of the first columne.

For Example.

♏ 22° 36′ is distant from ♈ 0, deg. 232, 36′.

And from ♎ 0, deg. 37, 24′. and from ♋ 0, deg. 142, 36′: which two last are conjunct arches.

And from the next *Æquinox* (which is ♎ 0) deg. 52, 36′, the complement of 37° 24′ shewed in the table.

And this is a *generall rule*, that if an arch tend from a point Eastward or according to the consequence of Signes, it is to be added to that point: but if it tend Westward or contrary to the Signes, it is to be subducted.

9. *Any two of these being given, to find out the other three.*

B, *The greatest Declination of the Sunne.*

BC, *The arch of the Ecliptic from the next Æquinox.*

CA,

CA, *The Declination of the Sunne in that place:*

BA, *The right ascension of that arch*: that is the arch of the *Æquinoctial* conterminating, or coming into the *Meridian* therewith.

C, *The angle which the Ecliptic maketh with the circle of Right Ascension.*

As if the Sunne be in ♌ 7° 25′: BC is 52° 35′: CA 18° 28′: BA 50° 10′: C 75° 12′.

Conſect. If the Sunnes greateſt Declination be reconed on the limb from the diameter, and the Index ſet therto: you ſhall have given both the Declination, and the Right Aſcenſion, for every degree of the Ecliptic.

10. *Any two of theſe being given, to find out the other three.*

B, *The complement of the Poles altitude.*

CA, *The Declination of the arch of the Ecliptic.*

BC, *The Amplitude Ortive.*

BA, *The Aſcenſional Difference.*

C, *The Angle which the circle of Right Aſcenſion maketh with the Horizon.*

As if the Pole be elevated 51° 32′: and the Sunne in ♌ 7° 25′, having Declination 18° 28′:

B is 38° 28′: CA 18° 28: BC 30° 36′5: BA 24° 51′:

C 55° 30. The greateſt Ampl: Ortive is 39° 52′.

Conſect:

Coroll: If the complement of the Poles altitude be reconed on the limb from the diameter, and the Index set thereto: you shall have given both the Amplitude Ortive, and the Adscensional Difference for every Declination adsigned.

Againe if the Index taken for a quadrant of the *Ecliptic*, be set at the greatest Ampl: Ort: reconed on the limb from the diameter; the arches of the great circles intercepted between the Index and the diameter, shall give the Ampl: Ort: for every degree of the *Ecliptic*.

11. *The Oblique Adscension of the Sunne*, that is the degree of the Æquinoctiall, rising together with the Sunne upon an Oblique Horizon, *is thus found.*

First seek the Right Adscension of the Sun from ♈ 0, by sect: 8 & 9, and from the next Æquinox: seek the Adscensional Difference, by sect. 10. Then if the Sunne be in any of these Northerne signes ♈ ♉ ♊ ♋ ♌ ♍, subduct the Adscensional Difference out of the Right Adscension. But if the Sunne be in any of the Southerne Signes ♎ ♏ ♐ ♑ ♒ ♓, Adde the Adscensional Difference to the Right Adscension. So shall you have the Suns Oblique Ascension desired.

But for the *Suns oblique Descension*, you must

must doe the quite contrary.

12. The Adscensional difference being converted into howers by sect. 7, sheweth how much before or after VI the Sunn riseth and setteth every day. Whereby also the length of the day is given.

The Rules following, though they also have divers *other* uses, yet they *specially conduce to the setting a figure of the Twelve celestial howses.*

13. *To find the Right Adscension of Medium Cœli* (*MC*) that is the point of the Æquinoctial culminant in the Meridian at any time assigned.

Convert the whole time from Noon last past into degrees of the Æquinoctial, by sect. 7. And thereto adde the Right Adscension of the Sunne.

14. If to the Right Adscension of MC found, you adde 90 degrees: you shall have the degree of the Æquinoctial Orient, or rising upon the East Horizon: the 180th degree from which is the degree of the *Æquinoctial* Occident or setting in the West. And *these* are *the Right Adscensions of the first and 7th howses.*

But for the *degree of the Æquinoctial in the rest of the howses above the Horizon*: To the Right

Right Adſcenſion of MC, adde 30 for the 11th houſe; and 60 for the 12th houſe.

Againe out of the Right Adſcenſion of MC, ſubduct 30 for the 9th houſe; and 60 for the 8th houſe.

15. In finding out the *Cuſpides of the houſes*, that is, what degree of the Ecliptic is entering upon the circle of every howſe; theſe three things are to be conſidered: *firſt* the lower angle of the Ecliptic with the Meridian towards the Horizon: *Secondly* the arch of the *Ecliptic* between the *Meridian* and the Horizon: *Thirdly* which of the Æquinoctiall points is above the Horizon, and on which ſide of the Meridian, Eaſt or Weſt. *The caſes hereof are theſe fower.*

1. Untill 90 degr: of the Æquinoctial culminating, the ſaid lower angle is acute on the Weſt ſide: The arch of the Ecliptic between the *Meridian* and the Weſt *Horizon*, leſſe then 90: And ♈ on the Weſt ſide.

2. Untill 180 degr: culminant, the ſaid lower angle is acute on the Eaſt ſide: The arch of the *Ecliptic* between the *Meridian* and the Eaſt *Horizon* leſſe then 90: And ♎ on the Eaſt ſide.

3. Untill 270 degr: culminant, the ſaid lower angle is acute on the Eaſt ſide: The arch of the *Ecliptic* between the *Meridian* and the

the East *Horizon* lesse then 90: And ☊ on the West side.

4. Untill 360 degr: culminant, the said lower angle is acute on the West side: The arch of the *Ecliptic* between the *Meridian* and the West *Horizon* lesse then 90: And ♈ on the East side.

These fower cases must be diligently considered.

16. *To find out the Cuspis of the Tenth house, called Cor Cœli.*

B, is the greatest Declination of the Sunne.

B A, The arch of the *Æquinoctial* between M C, and the next *Æquinox.*

B C, The arch of the *Ecliptic* conterminating with B A in the *Meridian*: which is the Cuspis of the tenth house sought.

C A, The Declination of that arch, or of Cor Cœli.

C, The lower angle of the *Ecliptic* with the *Meridian*: of which, as also of the conjunct obtuse, there is great use.

17. *To find out the Adscendent point of the Ecliptic, or the Cuspis of the First house, called Horoscopus.*

If the Declination of Cor Cœli be Northern, adde it to the complement of the Poles altitude; but if it be Southern, subduct it: So shall the summe or the Difference be the

altitude

altitude of Cor Cœli above the *Horizon*. Then upon the Instrument compute thus,

B A, the altitude of Cor Cœli.

B, the acute angle of the *Ecliptic* with the *Meridian*, found by sect. 16.

B C, the arch of the *Ecliptic* between Cor Cœli and the *Horizon:* which giveth the cuspis of the first or 9th house; according to the cases in sect. 15.

C A, The complement of the Amplitude Ortive of the Ascendent degree; which is also the Azumich of the 90us gradus of the *Ecliptic*, from the *Meridian*.

C, The angle of the *Ecliptic* with the *Horizon*; which is also the altitude of the 90us gradus.

18. *To find the Cuspides of the* 12th *and* 11th *howses; and also of the* 8 *&* 9 *howses.*

First the angle which each house-circle maketh with the Meridian, is to be sought, thus,

B A, The complement of the Poles altitude.

C A, The Arch of the *Æquinoctial*, degr: 30 for the 11th and 9th houses; but degr: 60 for the 12th and 8th houses.

These being set upon the instrument, will shew the angle B, which each of these houses maketh with the *Meridian* respectively. As at the altitude of the Pole 51° 32', the angle of the 11 & 9th house shall evermore be 42° 52':

and

and of the 12 and 8 house 70° 14'. Then proceed thus,

B, The angle of the house whose Cuspis you seek.

BC, The altitude of Cor Cœli above the *Horizon*, by sect. 17.

CA, The altitude of Cor Cœli above that house.

C, An angle evermore to be subducted out of the angle of the Ecliptic with the *Meridian*, on that side the house is, be it acute or obtuse, according to sect. 15 & 16. keep the Remaines in mind, for a new work wherein

B, Shall be the Remaines kept.

BA, The altitude of Cor Cœli above that house.

BC, The distance of the cuspis of that house from Cor Cœli: to be added to Cor Cœli or subducted out of it, according as that house is either on the East or West side of the *Meridian*.

For the more clear explanation of that which hath been delivered touching the erecting of a figure of the twelve cœlestial houses, I think it fit to set down the manner of working in this Example. Suppose a figure to be erected for 21 minuts after 8 a clock in the morning, or Hor: 20° 21', from the noon before; the Sunne being in ♎ 22° 59', that is having Longitude 202°

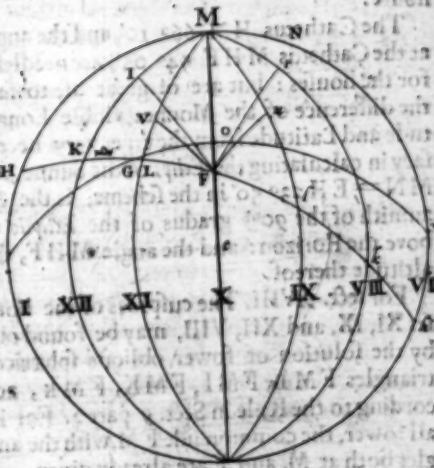
59′ 5 at the height of the Pole 51° 32′.

Hor: 20 21′ converted into degrees of the *Æquinoctial*, by sect. 7, will give 305° 15′. And the Right ascension of 202° 59′ by sect. 8 & 9, will be 201° 15′. for the R A of 22° 59′ is 21° 15′.

Then by Sect. 13, say 305° 15′ + 201° 15′ is 506° 30′, or (by subducting 360) 146° 30′, for the R A of M C, which is the point O in the Scheme: wherein the outer limbe is the *Horizon*. M e R the *Meridian*. E O W the *Æquinoctial*. H G F h the *Ecliptic*. The point F the cuspis of X house. and the intersection G, by case 2, sect. 15, is ♎0, because 146° 30′ falleth between 90 and 180.

For sect. XVI. In the rect. tri. G O F (that is B A C keeping the order of the letters) the angle G is 23° 30′. & the base G O is (180—146° 30′) 33° 30′. By which given you shall find the Cathetus F O 13° 30′. the hypotenusa G F 35° 43′. which being taken out of 180 (or ♎0) leaveth 144° 17′ of the *Ecliptic*, that is ♌ 24° 17′, for the cuspis of X house.

And lastly the acute angle F at the cathetus 70° 35′, is to be kept in mind, together with its conjunct 109° 25′.



For Sect. XVII. In the rect. triangle FMH, (that is BAC) the base F M is (38° 28′+13° 30′) 51° 58′. the acute angle F at the base 70° 35′. By which given parts shall find the hypotenusa F H 74° 19′: which being added to the point F 144° 17′, maketh up 219° 36′ of the Ecliptic, that is ♏ 9° 36′ in

for the Adſcendent point H, the cuſpis of 1
houſe.

The Cathetus H M 66° 10′, and the angle at the Cathetus M H F 54° 05′; are needleſſe for the houſes: but are of great uſe to find the difference of the Moones viſible Longitude and Latitude from the true; very neceſſary in calculating the *Eclips* of the Sunne. for M N = E H 23° 50′ in the ſcheme, is the Azumith of the 90ᵘᵃ gradus of the *Ecliptic* above the Horizon: and the angle M H F, the altitude thereof.

For ſect. XVIII. The cuſpides of the houſes XI, IX, and XII, VIII, may be found out by the ſolution of fower oblique ſphærical triangles F M L, F M I, F M K, F M k, according to the Rule in Sect. 9 part 2. For in all fower, the common ſide F M, with the angles both at M, and F are already given.

Or rather they may be divided every one into two rectangled triangles by the Cathetl F V, F u. and F I, F i. Then by the common hypotenuſa F M 51° 58′, and the angles M at the baſe, 42° 52′, at 70° 14′, given, you may find out the reſpective cathetus: and the reſpective partial angle F, at the cathetus towards the *Meridian*: which being ſubducted out of his own total angle F, will leave another rect. tri: on the other ſide of the cathe-

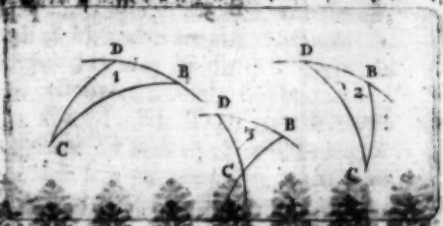
tus

tus towards the Ecliptic; in which is given the cathetus last found out, for the Base, and the remaining angle at F, for the angle at the Base; to find out the hypotenusa agreeable; which is the arch of the *Ecliptic* between F the cuspis of X house, and the circle of the proper house, that is the arch F L, F l, or F K, F k.

This may suffice for the direction of any ingenious practizer; and for the First part.

THE SECOND PART,
Of oblique-angled Spherical triangles.

1. IN an Oblique-angled ſpherical triangle BCD, five of the circular parts come into the account at once; namely one ſide BD, and the two oblique angles B & D; and the two other ſides DC & BC; the third angle C, oppoſit to BD, is not here inquired, but only the places of the angular points C, both for DC and BC. In



the firſt of theſe three ſchemes the inner angle D is

D is obtuſe, and B acute: in the ſecond ſcheme the inner angle D is acute, and B obtuſe: in the third ſcheme both the inner angles D and B are acute.

2. The ſide BD is evermore underſtood to be given. And in every operation, the firſt thing to be done is to open the legs of the Index at the widneſſe of BD, and there to ſcrew them faſt. Alſo I call the two legs of the Index, the leg or Index B, and the leg or Index D: as they are noted with thoſe letters.

3. The angles D and B of every triangle propoſed, are reconed upon the diameter from the limb, by the great circles; namely the angle B, and the conjunct of the angle D, from ♋; but the angle D, and the conjunct of the angle B, from ♑. And the ſides DG and BC, are to be reconed by the paralles; namely the ſides themſelves from the Pole; or their complements and exceſſes from the diameter. Note that of a triangle the conjunct angle, or the outer angle, is all one.

4. *Two ſides of an Oblique-angled ſpherical triangle, whereof one is* BD, *with the angle intercepted, being given, to find out at one work the third ſide; and the other angle at* BD. Here are theſe two varieties

Given

Given { DC & D / BC & B } to find out { BC & B. / DC & D.

5. The Rule. If the angles D and B be reconed in the upper semicircle of the Instrument at the Pole N; and the angle D be obtuse, or B acute; the point C shall be taken in the former semicircle towards ♋: but if the angle D be acute, or B obtuse; the point C shal be taken in the later semicircle towards ♑, & contrariwise for the lower Pole S. Then upon the Diameter recon the angle given, D or B, either by it self, or by its conjunct, according as was taught in Sect. 3. And from the end of that reconing imagine an arch of a great circle aptly traced, untill it meet with the Parallel of the complement of the respective side given, DC or BC: For that point of concurrence shall be the point C, for the given side, either DC or BC. To this point C thus found apply the Index noted with the proper leter of the angle given, D or B; and mark the distance of it from the center: For this distance being exactly taken on the other Index, shall in the Instrument give the true place of the point C sought: which being aptly æstimated, will upon the Diameter shew the angle sought, either B, or the conjunct of D, from ♋: or els the conjunct of B, or the angle D, from ♑. And it will also

shew, upon the limb the complement of the side sought.

Note here that in Astronomical operations, the angular point B of the triangle, is for the Pole of the World and the point D is either for the Pole of the *Ecliptic*; and then the Limb is the Solstitial Colure; and the side DC the circle of the starrs Longitude; and the side BC the circle of its Right Ascension; and the side BD the Sunnes greatest Declination. Or els the point D is for the Pole of the Horizon, and then the limb is the *Meridian* of the place; and the side DC the Azunith; and the side BC the Horarie circle; and the side BD the complement of the Poles elevation.

6. The Rule being thus plainly set down, we will come to shew the practice of it in some examples.

Example 1, wherein the angle D is obtuse, and B acute. Suppose a starre having Longitude ♉ 24° 50′, or els ♎ 5° 10′, both which places are equally distant from ♋ or ♑; and latitude Northern 17° 30′: it is required to find out at one operation, the Right Ascension, and Declination of both those starrs severally.

This, and such like questions, are performed by the resolution of a sphærical triangle

BCD

BCD: in which BD is 23° 30'; and DC the complement of 17° 30'; and the inner angle D (by the table in Sect. 8. part 1) is 144° 50', the distance of either starres Longitude from ♈; the conjunct whereof is 35° 10', the distance from ♋: according to this distance trace a great circle till it meet with the parallel 17° 30'. Then unto the point of concurrence apply the proper Index D; and it will cut therein 56° 43' from the center: recon the very same number on the other Index B: and it shall shew the great circle 33° 20' from ♋, for the inner angle B; and the parallel degr: 2 beneath the Diameter, for the excesse of the side BC. Wherefore (by tab. Sect. 8 part 1) the Right Ascension orcoming into the *Meridian*, of one starre, shall be with the degre of the *Æquinoctial* 56 40'; and of the other starre with 123° 20'. And of both the Declination is deg. 2 Southern.

This example being well understood and practiced, the rest will be easy.

Example II, wherein the angle D is acute, and B obtuse. Suppose a starre having Longitude ♌ 22° 22'; or ♍ 7° 38'; both which places are equally distant from ♈ or ♋: and the Latitude Northern deg: 18: it is required to find out the Right Ascension, and Declination of both those starres severally.

In

In the ſpherical triangle B C D here to be reſolved, are given the ſide B D 23° 30′: and D C the complement of deg. 18; and the inner angle D (by tab. in Sect. 2. part 1.) is 52° 22′ from ♈. Seek out the point of the proper Index D, at which the parallel deg. 18 interſecteth the great circle 52° 22′ from ♈. For the very ſame diſtance, which will be 40° 59′, taken on the other Index B, ſhall in the Inſtrument give the great circle 118° 30′ from ♎, for the inner angle B: and the parallel deg. 31, for the complement of the ſide B C. Wherefore (by tab. ſect. 2 part 1.) the Right Aſcenſion of one ſtarre ſhall be 331° 30′, and of the other ſtarre 208° 30′. And of both the Declination is deg. 31 Northern.

Example III, wherein both angles D and B are acute. Suppoſe a ſtarre having Longitude ♓ 1° 44′, or ♓ 28° 16′, both which places are equally diſtant from ♈: And the Northern Latitude deg. 56. It is required to find out the Right Aſcenſion, and Declination, of both thoſe ſtars ſeverally.

In the ſpherical triangle B C D, here to be reſolved, are given, B D 23° 30′: and D C the complement of deg. 56: and the inner angle D (by tab. in Sect. 2 part 1.) 61° 44′, from ♈. Seek out the point of the proper

Index

Index D, at which the parallel deg. 56 intersecteth the great circle 61° 44′ from ♈. for the very same distance, which will be 60° 30′ taken on the other Index B, shall in the Instrument give the great circle 80° 2′ from ♋, for the inner angle B ♋ and the parallel deg. 60, for the complement of the side B C.

Wherefore (by tab. in sect. 8 part 1.) the Right Ascension of one starre shall be 90° 58′: and of the other starre 170° 2′. And of both the Declination is deg. 60 Northern.

Example IV., of Azumiths, and Horary circles. Suppose at a known elevation of the Pole, the Sunne, or a starre above the *Horizon*, to have its Azumith, or Horizontal distance from the South *Meridian* 58° 52′: and its Almicantarah, or Altitude above the *Horizon*, deg. 17. To find out at one operation, the Horary circle, and the Declination.

In the Sphærical triangle B C D to be resolved for this question, are given the side B D (the complement of the Poles elevation) 38° 30′: and D C the complement of deg. 17: and the outer or conjunct angle D 58° 52′.

Seek the point of the Index D, at which the parallel of deg. 17 intersecteth the great circle 58° 52′. For the very same distance, which will be 35° 4′, taken on the Index B, shall in the Instrument give the great circle

55° 58′ for the inner angle B; and the parallel deg: 4 beneath the Diameter, for the Exteſſe of the ſide B C.

Wherefore the Horarie circle ſhall be 55° 58′ from the ſouth *Meridian*: And the Declination deg. 4 Southern, or below the *Æquinoctial*.

7. A collection or view of the five circular parts, in the fower former examples, together with the diſtance of the points C from the center.

Examp	I	II	III	IV
B D	23 30	23 30	23 30	38 30
D C	72 30	72 00	34 00	72 00
D	144 30	52 22	61 44	56 52
B C	92 00	59 00	30 00	94 00
B	33 20	118 30	80 02	54 58
C C	56 43	40 59	60 30	35 04

8. By this which hath been ſhewed, that the point C, both of the ſides D C and B C, are evermore æqually diſtant from the center, there is opened unto us a way, whereby *any two of thoſe five circular parts, together with the ſide B D, being given, the other two may at one operation be found out.*

Herein are theſe fower varieties.

Given

Given { DC & BC / DC & B / D & BC / D and B } to find out { D & B. / D & BC. / DC & B. / DC & BC }

9. The Rule. In the upper semicircle of the Instrument, If the angle D be obtuse, or if the side D C be much lesser then B C, the point C shall be taken in the former quadrant towards ♋: but if D be acute, or DC much greater then BC, the point C shall be taken in the later quadrant toward ♑. And contrarywise in the under semicircle. And in both if the angles D and B be acute, or if the sides D C and BC differ but little in length, the true point C sought shall fall neer the Axis toward ♑, within a space equall to the Index opened at the widnesse of B D.

Set therefore the Index thus opened, so that the two points C may fall within it: And among the great circles and parallels, or both (according to 8. 3, 4, 5; and as reason shall direct) recon the two circular parts given, tracing them proportionally, with a pin in each hand, untill they both shall exactly meet with the proper legges of the Index at an equal distance from the center, as neer to it

it as possible may be: So have you in the Instrument the true places of both the points C; and thereby the two circular parts sought, respectively to each point C, both of DC and BG.

I shall not need to deliver any examples of this Rule, having already in sect. 7, 8, set down fower Examples calculated, and in each Example fower varieties: so that in all you have Sixteen examples to practice upon at leysure, and to make you more expedite and familiar in this way of operation.

10. And note here, that if any point C falleth either among the parallels nere the Diameter, or among the great circles neer the Limb, where they lye almost æqui-distant, ther may be some uncertainty in finding the true points exactly: the remedying whereof requireth a more solertious diligence in the computer. But if yet any difficulty remaine, you may, to satisfy your self, distinguish your oblique triangle given, into two Right-angled triangles, by a Cathetus aptly drawn, and so resolve them by the first part of this treatise: Or els by the Rules of Trigonometry.

11. *If the three angles of an Oblique spherical triangle be given,* they are to be converted into to their opposit sides: the two lesser angles and

and sides being kept as they are; but the greatest angle with the side opposit to it, are to be changed into their conjuncts, that is, the complemens of them to a semicircle, which I note with this mark '. as supposing C to be the greatest angle;

B and D C
D and B C } shall be mutually taken one
C' and B D' } for the other.

The end of the second part, and of the whole Treatise. For which

Soli Deo sit omnis laus & gloria.

A M E N.